Roll No. .....

BCA–205(N)

B. C. A. (Second Semester) EXAMINATION, 2016

(New Course)

Paper Fifth

MATHEMATICS—II

Time : Three Hours] [*Maximum Marks : 75*

Note : Section A is compulsory. Attempt any *seven* questions from Section B and attempt any *one* question from Section C.

Inst.: The candidates are required to answer only in serial order. If there are many parts of a question, answer them in continuation.

Section—A 8 each

1. (a) If $u = (lx + my + nz)^2 - (x^2 + y^2 + z^2)$ and $l^2 + m^2 + n^2 = 1$, show that :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

(b) Find the equation of plane through (1, 1, –1) and (1, –1, 1) and perpendicular to the plane $x + 2y + 2z = 5$.

2. (a) Evaluate $\iint xy(x+y)\,dx\,dy$ over the area between $y = x^2$ and $y = x$.

(b) Discuss maximum and minimum of function $x^3 + y^3 - 6xy$.

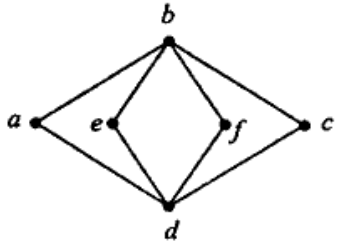
Section—B 6 each

3. Prove that $A - B = B' - A'$ where A and B are any *two* non-empty sets and A' denotes complement of set A.
4. Let :

$A = \{1, 2, 3, 4\}$ and $R = \{(1,2), (2,3), (3,4), (2,4)\}$.

Find transitive closure of R.

5. Find the equation of sphere through point (2, 1, 3) and the circle $x^2 + y^2 = 4, z = 0$.
6. Let $f : \mathbf{R} \to \mathbf{R}$ be defined by $f(x) = 1 + x^2$ and $g : \mathbf{R} \to \mathbf{R}$ by $g(x) = \log(x)$. Find (a) f o g (b) f o f, where **R** is set of real numbers.
7. If $y = f(x) = \left(\frac{2x-1}{5x-2}\right)$, prove that $f(y) = x$.
8. Define POSET. Draw Hasse diagram (A, /) where A = {3, 4, 12, 24, 48, 72} and the relation '/' is a/b if a divides b.
9. Show that the lattice L represented by diagram is complemented but not distributive.



10. Find direction cosines l, m, n of two line which are connected by relation :

$$l - 5m + 3n = 0$$

$$7l^2 + 5m^2 - 3n^2 = 0$$

11. If :

$$u = \sin^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$$

then show that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$$

12. The mid-points of sides of triangle are (1, 5, –1), (0, 4,–2) and (2, 3, 4). Find the vertices of triangle.

Section—C 17 each

13. (a) If $u = f(y - z, z - x, x - y)$ prove that :

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$$

(b) Find the points on $z^2 = xy + 1$ nearest to origin.

14. (a) Prove that if R is an equivalence relation on set A, then R^{-1} is also an equivalence relation on A.

(b) Let R be equivalence relation on set of integers Z as $R = \{(x, y) : x - y \text{ is divisible by } 3\}$. Prove that R is an equivalence relation.

15. (a) Find the points in which the line $\frac{x+1}{-1} = \frac{y-12}{5} = \frac{z-7}{2}$ cuts the surface $11x^2 - 5y^2 + z^2 = 0$.

(b) Find the volume of tetrahedron bounded by the co-ordinate planes and the plane $x + y + z = 1$.