BCA -105(N)

Roll No. :.................................

B.C.A. (First Semester)

EXAMINATION, Dec.-2015

(New Course)

Paper – Fifth

MATHEMATICS-I

[Time: Three Hours]   *[Maximum Marks: 75]*

Note : Attempt questions from all the questions at directed.

SECTION – A

Note : Attempt all the questions, each will carry 3 marks.

Short Answer Type Questions Only

1.   A   Show that the adjoint of

$$\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

is three times its transpose.

B.   Find the equation at the line joining $(1, 2)$ and $(3, 6)$ using determinants.

C.   If $f(x) = \begin{cases} \dfrac{\sin [x]}{[x]} & , \text{ if } [x] \neq 0 \\ 0 & , \text{ if } [x] = 0 \end{cases}$

Where $[x]$ denotes the greatest integer less than or equal to x, then find $\lim_{x \to 0} f(x)$.

[2]

D.   Show that $f(x)-x|x|$ is differentiable at $x=0$.

E.   Differentiate $\tan^{-1} \dfrac{x}{\sqrt{1-x^2}}$ with respect to $\cos^{-1}(2x^2 - 1)$.

F.   Evaluate $\displaystyle\int_0^\infty \dfrac{x^8(1-x^6)}{(1+x)^{24}}$

G.   Evaluate $\displaystyle\int \dfrac{dx}{x(x^n+1)}$

H.   Find the angle between the vectors $\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}$.

I.   If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 5\hat{i} + 5\hat{j} - \hat{k}$, find $(\vec{a} + \vec{b}) \times 2\vec{b}$ and $(\vec{a} + \vec{b}) \times 2\vec{b}$

SECTION – B

12 Marks Each Question

Long Answer Type Questions

(02 out of 04 questions)

Find the Eigen value and Eigen vector of the following matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$$

Expand $e^{a\sin^{-1}x}$ by Maclaurin's theorem and find the general term.

4.   (a)   Reduce the following matrix into normal form and hence find its rank

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

(b)   Prove that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$$

[3]

5. (a) Show that the following limit does not exist

$$\lim_{x \to 3} \frac{|x - 3|}{x - 3}$$

(b) Discuss the nature of discontinuity of the following function at the origin

$$f(x) = \begin{cases} \dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}} & \text{, when } x \neq 0 \\ 0 & \text{, when } x = 0 \end{cases}$$

SECTION – C

12 Marks Each Question

Long Answer Type Questions
(02 out of 04 questions)

6. (a) Discuss the maxima and minima of the function

$$f(x) = \frac{x}{1+x^2}$$

(b) Trace the curve
$$ay^2 = x^2(a-x)$$

7. (a) Evaluate $\int \sec^3 x \, dx$

(b) if $\ln = \int_0^{\pi/4} \tan^n x \, dx$,

show that $\ln + \ln{-2} = \dfrac{1}{n-1}$

8. (a) Evaluate $\int_0^\infty x^m e^{-ax^n} dx$, when m, n, a are positive constants.

(b) Evaluate $\int \cos x \cos 2x \cos 3x \, dx$

9. (a) If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors show that
$$[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$$

(b) Prove that
$$\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}.$$