Roll No...................

BCA-205 (N)
B.C.A. (Semester-II) Examination-2014
(New Course)
Paper: Fifth
Mathematics-II

Time: Three Hours]   [Maximum Marks: **75**

Note: Section A is compulsory. Attempt any seven questions from Section B and attempt any one question from Section C.

Section-A

Note: All questions are compulsory. Each question carries 8 marks.   (8x2=16)

1. (a) If $U = \log(x^3 + y^3 + z^3 - 3xyz)$,

     Show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 U = \frac{-9}{(x+y+z)^2}$$

   (b) Evaluate $\iint (x^2 + y^2)dxdy$ over the region in the positive quadrant for which $x + y \leq 1$

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2. (a) Define Lattice. Show that the set L of all factors of 24 under divisibility forms a lattice.

   (b) State and prove Demogram's laws.

Section-B
(Short Answer Type Questions)

Note: Attempt any seven questions. Each question carries 6 marks.   (7x6=42)

3. Prove that if $f: A \to B$ is one-one on to mapping then $f^{-1}: B \to A$ will be one-one onto mapping.

4. Consider the set NxN the set of ordered pairs of natural numbers. Let R be a relation in NxN which is defined by $(a,b)R(c,d)$ iff $ad = bc$. Prove that R is an equivalence relation.

5. Show that dual of a lattice is a lattice.

6. Examine $f(x,y) = xy + \frac{a^3}{x} + \frac{a^3}{y}$ for maximum and minimum value.

7. The projections of a line on axis are 5,10,10. Find the length and direction cosines.

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8. Evaluate- $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dxdy}{1+x^2+y^2}$

9. Find the equation to the sphere through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ and the point $(1,2,3)$

10. Change the order of integration in $\int_0^1 \int_{\sqrt{x}}^1 e^{x/y} \, dx \, dy$ and hence find its value.

11. In a class of 25 students, 12 have taken mathematics, 8 have taken mathematics but not Biology. Find the number of students who have taken Mathematics and Biology and those who taken Biology but not Mathematics.

12. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, then

Show that $f\left(\frac{2x}{1+x^2}\right) = 2f(x)$

Section-C
(Long Answer Type Questions)

Note: Attempt any one question. Each question carries 17 marks. (17x1=17)

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13. Find the image of the point $(1, 3, 4)$ in the plane $2x - y + z + 3 = 0$

14. If $\theta = t^n e^{-r^2/4t}$, what value of n will make $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \theta}{\partial r}\right) = \frac{\partial \theta}{\partial t}$?

15. (a) If R and S be two equivalence relation in A. Then prove that $R \cap S$ be also an equivalence relation in A.

(b) If $f(x)$ is defined on $[0,1]$ by the rule
$$f(x) = \begin{cases} x & if \ x \in Q \\ 1-x & if \ x \notin Q \end{cases}$$

Prove that $f \ of \ (x) = x, \forall x \in [0,1]$

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