Roll No. ....

BCA–405(N)

B. C. A. (Fourth Semester)

EXAMINATION, May, 2019

(New Course)

Paper Fifth

MATHEMATICS–III

Time : Three Hours] [*Maximum Marks : 75*

Note : Attempt questions from all Sections as directed.

Inst. : The candidates are required to answer only in serial order. If there are many parts of a question, answer them in continuation.

Section—A

(Short Answer Type Questions)

Note : All questions are compulsory. Each question carries 3 marks.

1. (A) Find real numbers x and y, if :

$$x + iy = \frac{2-3i}{7+4i}$$

(B) Define monotonic sequence and explain it with example.

(C) If $\vec{r} = (\cos nt)\hat{i} + (\sin nt)\hat{j}$, then show that :

$$\vec{r} \cdot \frac{d\vec{r}}{dt} = 0$$

(D) If $\vec{F} = x^2 z\hat{i} - 2y^3 z^2\hat{j} + xy^2 z\hat{k}$, find $\operatorname{div}\vec{F}$ at $(1, -1, 1)$.

(E) Simplify $\overline{(6+i)} + (7-i)^2 - (3-4i^2)$ and write in the form $a + ib$.

(F) Find the half-range cosine series for $f(n) = 1$ in $0 < x < \pi$.

(G) Solve the differential equation :

$$\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$$

(H) Solve :

$$(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$$

(I) Solve :

$$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = 0$$

Section—B

(Long Answer Type Questions)

Note : Attempt any *two* questions. Each question carries 12 marks.

2. (a) Find all value of $(1+i)^{\frac{1}{3}}$.

   (b) Solve the equation :

$$\tan^{-1}\left(\frac{x+1}{x-1}\right)+\tan^{-1}\left(\frac{x-1}{x}\right)=\tan^{-1}(-7)$$

3. (a) Prove that :

$$\lim\left[\left(\frac{2}{1}\right)\cdot\left(\frac{3}{2}\right)^2\cdot\left(\frac{4}{3}\right)^3\ldots\left(\frac{n+1}{n}\right)^n\right]^{\frac{1}{n}}=e$$

   (b) Discuss the convergence of the series :

$$\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\ldots\ldots$$

4. Find the directional derivative of :

$$f(x,y,z)=x^2yz+4xz^2$$

at the point (1, –2, –1) in the direction of the vector $2\hat{i}-\hat{j}-2\hat{k}$.

5. Test for convergence the series :

$$\sum_{n=1}^{\infty}\frac{3.6.9\ldots\ldots3n}{7.10.13\ldots\ldots(3n+4)}x^n,\ (x>0)$$

Section—C

(Long Answer Type Questions)

Note : Attempt any *two* questions. Each question carries 12 marks.

6. Obtain Fourier's series of $f(x)=x\sin x$ in the interval $(-\pi,\pi)$. Hence deduce that :

$$\frac{\pi}{4}=\frac{1}{2}+\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\ldots\ldots$$

7. (a) Solve :

$$x\frac{dy}{dx}+y=y^2\log x$$

   (b) Solve :

$$(x^2-ay)\,dx-(ax-y^2)\,dy=0$$

8. Solve :

$$\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+3\frac{dy}{dx}-y=xe^x+e^x$$

9. Solve :

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=8x^2e^{2x}\sin 2x$$